

[tex114] BE gas in \mathcal{D} dimensions II: isochore

(a) From the fundamental thermodynamic relations for the Bose-Einstein gas in \mathcal{D} dimensions (see [tsc14]), derive the following parametric expression for the isochore at $T \geq T_c$:

$$\frac{p}{p_v} = \frac{g_{\mathcal{D}/2+1}(z)}{[g_{\mathcal{D}/2}(z)]^{2/\mathcal{D}+1}}, \quad \frac{T}{T_v} = [g_{\mathcal{D}/2}(z)]^{-2/\mathcal{D}},$$

where $k_B T_v = \Lambda v^{-2/\mathcal{D}}$ and $p_v = \Lambda v^{-2/\mathcal{D}+1}$ with $\Lambda \doteq h^2/2\pi m$ are convenient reference values.

(b) Calculate the leading correction to the Maxwell-Boltzmann result at high temperature. (c) Calculate the exact dependence of p/p_v on T/T_v at $T \leq T_c$ in $\mathcal{D} > 2$. Show that this result also holds asymptotically for $T \ll T_v$ in dimensions $\mathcal{D} = 1$ and $\mathcal{D} = 2$.

Solution: