## [tex113] BE gas in $\mathcal{D}$ dimensions I: fundamental relations

From the expressions for the grand potential and the density of energy levels of an ideal Bose-Einstein gas in $\mathcal{D}$ dimensions and confined to a box of volume $V = L^{\mathcal{D}}$ with rigid walls,

$$\Omega(T, V, \mu) = k_B T \sum_k \ln(1 - z e^{-\beta \epsilon_k}), \qquad D(\epsilon) = \frac{V}{\Gamma(\mathcal{D}/2)} \left(\frac{m}{2\pi\hbar^2}\right)^{\mathcal{D}/2} \epsilon^{\mathcal{D}/2-1},$$

derive the fundamental thermodynamic relations at fugacity $z < 1$ in terms of the Bose-Einstein functions $g_n(z)$ and the thermal wavelength $\lambda_T = \sqrt{h^2/2\pi m k_B T}$ as follows:

$$\frac{pV}{k_B T} = \frac{V}{\lambda_T^{\mathcal{D}}} g_{\mathcal{D}/2+1}(z), \quad \mathcal{N} = \frac{V}{\lambda_T^{\mathcal{D}}} g_{\mathcal{D}/2}(z), \quad U = \frac{\mathcal{D}}{2} k_B T \frac{V}{\lambda_T^{\mathcal{D}}} g_{\mathcal{D}/2+1}(z).$$

**Solution:**