

[tex110] Occupation number fluctuations

Consider an ideal quantum gas specified by the grand partition function Z . Start from the expressions,

$$\langle n_k \rangle = \frac{1}{e^{\beta(\epsilon_k - \mu)} + a}, \quad \langle n_k^2 \rangle - \langle n_k \rangle^2 = -\beta^{-1} \frac{\partial \langle n_k \rangle}{\partial \epsilon_k},$$

explained in [tln64], where $a = +1, 0, -1$ represent the FD, MB, and BE cases, respectively, to derive the following result for the relative fluctuations in the occupation numbers:

$$\frac{\langle n_k^2 \rangle - \langle n_k \rangle^2}{\langle n_k \rangle^2} = \frac{1}{\langle n_k \rangle} - a.$$

Note that in the BE (FD) statistics, these fluctuations are enhanced (suppressed) relative to those in the MB statistics.

Solution: