[tex106] Ideal-gas engine with two-step cycle I

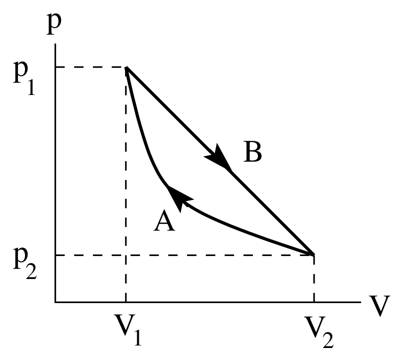
Consider the two-step cycle for a classical ideal gas $[pV = Nk_BT, C_V = \alpha Nk_B, \gamma \doteq C_p/C_V = (\alpha+1)/\alpha]$ as shown. The first step (A) is an adiabatic compression and the second step (B) an expansion along a straight line segment in the (V,p)-plane.
(a) Show that the difference in internal energy $\Delta U \doteq U_1 - U_2$ is determined by the expression

$$\frac{\Delta U}{p_1 V_1} = \alpha \left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}\right].$$

(b) Show that the heat transfer δQ between system and environment during a volume increase from V to $V + \delta V$ along the straight line segment is given by the expression

$$\frac{\delta Q}{p_1 V_1} = \left[(1+\alpha)(1+\sigma) - (1+2\alpha)\sigma\,\frac{V}{V_1}\right]\frac{dV}{V_1}, \quad \sigma = \frac{1-(V_1/V_2)^\gamma}{V_2/V_1 - 1}.$$

(c) Show that along the straight-line segment the system absorbs heat if $V_1 < V < V_c$ and expels heat if $V_c < V < V_2$, where $V_c/V_1 = [(1+\alpha)(1+\sigma)]/[(1+2\alpha)\sigma]$.



Solution: