[tex100] FD gas in $\mathcal{D}$ dimensions: heat capacity at high temperature

The internal energy of the ideal Fermi-Dirac gas in $\mathcal{D}$ dimensions is given by the expression,

$$U = \mathcal{N} k_B T \, \frac{\mathcal{D}}{2} \frac{f_{\mathcal{D}/2+1}(z)}{f_{\mathcal{D}/2}(z)}.$$

(a) Use this result to derive the following expression for the heat capacity $C_V = (\partial U/\partial T)_{V\mathcal{N}}$:

$$\frac{C_V}{\mathcal{N} k_B} = \left(\frac{\mathcal{D}}{2} + \frac{\mathcal{D}^2}{4}\right) \frac{f_{\mathcal{D}/2+1}(z)}{f_{\mathcal{D}/2}(z)} - \frac{\mathcal{D}^2}{4} \frac{f'_{\mathcal{D}/2+1}(z)}{f'_{\mathcal{D}/2}(z)}.$$

Use the derivative $\partial/\partial T$ of the result $f_{\mathcal{D}/2}(z) = \mathcal{N}\lambda_T^{\mathcal{D}}/gV$ with $V = L^{\mathcal{D}}$ to calculate any occurrence of $(\partial z/\partial T)_{V\mathcal{N}}$ in the derivation. Use the recursion relation $z f'_n(z) = f_{n-1}(z)$ for $n \geq 1$ to further simplify the results pertaining to $\mathcal{D} \geq 2$. (b) Infer from this result the leading correction to the Maxwell-Boltzmann result, $C_V = (\mathcal{D}/2)\mathcal{N} k_B$, at high temperature.

Solution: