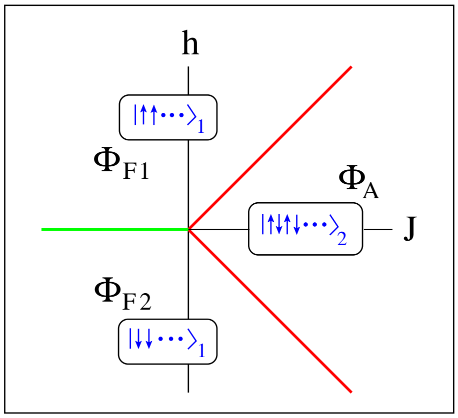
$\mathbf{T = 0}$ phase diagrams of Ising chains [tln96]

Spin quantum number $s = \frac{1}{2}$:

The gound state of the 1D $s = \frac{1}{2}$ Ising model with nearest-neighbor (nn) coupling, specifically its magnetization, its periodicity, and its degeneracy, depends on the coupling strength J and the external magnetic field h.

$$\mathcal{H} = \sum_{l=1}^{N} \left[J S_l^z S_{l+1}^z - h S_l^z \right] \quad : \; s = \tfrac{1}{2}.$$



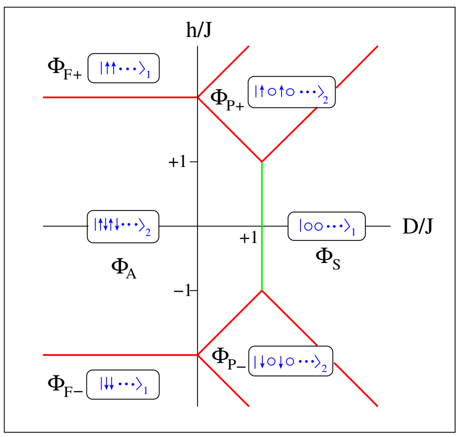
The (J, h)-plane shows three phases divided by three phase boundaries.

- Aligned (anti-aligned) nn spins are favored if $J < 0$ ($J > 0$).
- Two phases are ferromagnetic and one is antiferromagnetic.
- The subscript of each product vector denotes its periodicity in units of the lattice spacing.
- The ground state on the phase boundary at $h = 0$ and $J < 0$ is a state with twofold degeneracy.
- The ground state on phase boundaries at $h = \pm J$ is a highly degenerate state. It includes vectors of many different periodicities.
- The ground state is also named *physical vacuum*.
- The physical vacuum of each phase can be used as a reference state (*pseudo-vacuum*) for a set of particles that generate all eigenstates.
- It is possible to generate sets of particles that have definite activation energies, but no interaction energies – an idea developed in [tsc22].
- If the pseudo-vacuum coincides with the physical vacuum then all particle activation energies are non-negative.

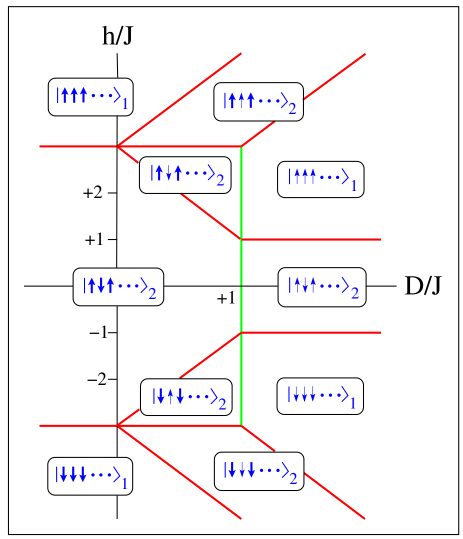
Spin quantum number $s = 1$ and $s = \frac{3}{2}$:

In the Hamiltonian of the Ising chain with $s > \frac{1}{2}$ an on-site term with coefficient D is added.[1] Here we consider only antiferromagnetic nn coupling ($J > 0$) and explore the parameter-plane $(D/J, h/J)$.

$$\mathcal{H} = \sum_{l=1}^{N} \left[J S_l^z S_{l+1}^z + D(S_l^z)^2 - h S_l^z \right] \quad : \; s > \tfrac{1}{2}, \quad J > 0.$$



$s = 1$



$s = 3/2$

– For $s = 1$ we use symbols $\uparrow, \odot, \downarrow$ for $S_z = +1, 0, -1$, respectively. For $s = \frac{3}{2}$ we use $\boldsymbol{\uparrow}, \uparrow, \downarrow, \boldsymbol{\downarrow}$ for $S_z = +\frac{3}{2}, +\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$, respectively.

– The distinct phases are now more numerous. The maximum periodicity is still two. Higher degeneracies are realized at the phase boundaries.

– Note the spin-flip relation between the ordering above and below the horizontal axis.

– At $h = 0$ (and $J > 0$) there is now more than one phase.

– A new feature is the presence of plateau phases with nonzero and non-saturated magnetization.

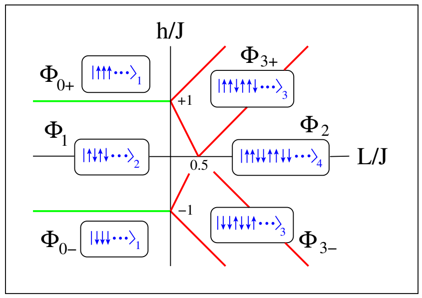
– The ground state of each phase (physical vacuum) can be employed as the reference state (pseudo-vacuum) for a set of particles populating it.

– The entire spectrum of the Ising chain can be generated by configurations of such particles [tsc22].

[1] The same term is a mere constant for $s = \frac{1}{2}$.

Nearest and next-nearest-neighbor couplings:

$$\mathcal{H} = \sum_{l=1}^{N} \left[J S_l^z S_{l+1}^z + L S_l^z S_{l+2}^z - h S_l^z \right] \quad : \; s = \tfrac{1}{2}, \quad J > 0.$$



– The addition of a next-nearest-neighbor coupling enriches the phase diagram considerably.

– There are now phases with spatial periodicity $p = 1, 2, 3, 4$.

– There are again two plateau phases, here with periodicity $p = 3$.

– The plateau phases found here and in the previous case satisfy the OYA condition[2] for the existence of such phases:

$$p(s - m) = \text{integer}$$

s: spin quantum number

m: magnetization per lattice site

p: periodicity of state

– The plateau phase in this application has $s = \frac{1}{2}$, $m = \frac{1}{6}$, and $p = 3$.

– The plateau phases in the previous application have $s = 1$, $m = \frac{1}{2}$, $p = 2$ and $s = \frac{3}{2}$, $m = 1$, $p = 2$.

– It is again possible to declare each of the physical vacua pertaining to one of the six phases as the pseudo-vacuum for a set of particles that generate the entire spectrum of the model in all six parameter regimes.

The main takeaway here is that the combination of spin coupling and external field can stabilize very diverse ordering patterns. There are no correlated quantum fluctuations in each product ground state.

Spin chain models with more complex ground states that include correlated quantum fluctuations will be discussed elsewhere.

[2]M. Oshikawa, M. Yamanaka, and I. Affleck, Phys. Rev. Lett. 78, 1984 (1997).