Blackbody radiation [tln68]

Electromagnetic radiation inside cavity in thermal equilibrium at temperature T. Grandcanonical ensemble of photons ($\epsilon = \hbar\omega = cp$, $\mathbf{p} = \hbar\mathbf{k}$, spin $s = 1$, bosonic, purely transverse).

Density of states: $D(\epsilon) = g\,\dfrac{4\pi V}{h^3c^3}\,\epsilon^2$ with $g = 2$ independent polarizations.

Average occupation number: $\langle n_\epsilon\rangle_{BE} = \dfrac{1}{e^{\beta\epsilon}-1}$.

Number of photons with energies between ϵ and $\epsilon + d\epsilon$:

$$dN(\epsilon) = \langle n_\epsilon\rangle_{BE} D(\epsilon)d\epsilon = \frac{8\pi V\epsilon^2}{h^3c^3}\frac{1}{e^{\beta\epsilon}-1}d\epsilon.$$

Spectral density inside cavity: [use $dN(\epsilon) = V dn(\omega)$ and $\epsilon = \hbar\omega$]:

$$\frac{dn(\omega)}{d\omega} = \frac{\hbar}{V}\frac{dN(\epsilon)}{d\epsilon} = \frac{\omega^2}{\pi^2c^3}\frac{1}{e^{\beta\hbar\omega}-1}.$$

Spectral energy density inside cavity: $du = \hbar\omega dn = \rho(\omega)d\omega$.

$$\rho(\omega)d\omega = \frac{\omega^2}{\pi^2c^3}\frac{\hbar\omega}{e^{\beta\hbar\omega}-1}d\omega = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\beta h\nu}-1}d\nu.$$

Rate (per unit area) at which particles with (average) speed c escape from cavity through small opening [tex62]: $dN/dt = \frac{1}{4}(N/V)c$.

Spectral density of radiation: $R(\omega) = \dfrac{c}{4}\dfrac{dn(\omega)}{d\omega} = \dfrac{\omega^2}{4\pi^2c^2}\dfrac{1}{e^{\beta\hbar\omega}-1}$.

Spectral energy density of radiation:

$$Q(\omega) = \hbar\omega R(\omega) = \frac{\omega^2}{4\pi^2c^2}\frac{\hbar\omega}{e^{\beta\hbar\omega}-1} \quad \text{(Planck radiation law)}.$$

High frequencies: ultrarelativistic MB particles [use $\langle n_\epsilon\rangle_{MB} = e^{-\beta\epsilon}$]:

$$Q(\omega) = \frac{\hbar\omega^3}{4\pi^2c^2}\,e^{-\beta\omega} \quad \text{(Wien radiation law)}.$$

Low frequencies: equipartition law applied to electromagnetic modes:

$$Q(\omega) = \frac{k_BT\omega^2}{4\pi^2c^2} \quad \text{(Rayleigh$-$Jeans radiation law)}.$$