

### [gex97] Inertia tensor from rotational kinetic energy of rigid body

Consider a rigid body in purely rotational motion with angular velocity  $\boldsymbol{\omega}$  about some instantaneous axis. The (instantaneous) kinetic energy can then be constructed from the expression,

$$K = \frac{1}{2} \sum_{\alpha} m_{\alpha} (\boldsymbol{\omega} \times \mathbf{r}_{\alpha})^2, \quad (1)$$

where  $m_{\alpha}$  are infinitesimal mass elements at positions  $\mathbf{r}_{\alpha}$  in a coordinate system fixed to the rigid body with its origin on the axis of rotation. Extract from (1) the expression,

$$I_{ij} = \sum_{\alpha} m_{\alpha} [\delta_{ij} r_{\alpha}^2 - r_{\alpha i} r_{\alpha j}], \quad (2)$$

for the inertia tensor by transforming (1) into the form,

$$K = \frac{1}{2} \sum_{ij} I_{ij} \omega_i \omega_j. \quad (3)$$

**Solution:**