

[gex95] **Relations between error function and Fresnel integrals**

Establish the relations,

$$\begin{aligned}\operatorname{erf}(\sqrt{i}z) &= (1+i) \left[ \operatorname{C} \left( \sqrt{\frac{2}{\pi}} z \right) - i \operatorname{S} \left( \sqrt{\frac{2}{\pi}} z \right) \right], \\ \operatorname{erf}(\sqrt{-i}z) &= (1-i) \left[ \operatorname{C} \left( \sqrt{\frac{2}{\pi}} z \right) + i \operatorname{S} \left( \sqrt{\frac{2}{\pi}} z \right) \right],\end{aligned}$$

between the error function and the Fresnel integrals [gmd4B],

$$\operatorname{erf}(z) \doteq \frac{2}{\sqrt{\pi}} \int_0^z dt e^{-t^2}, \quad \operatorname{C}(z) \doteq \int_0^z dt \cos\left(\frac{\pi}{2} t^2\right), \quad \operatorname{S}(z) \doteq \int_0^z dt \sin\left(\frac{\pi}{2} t^2\right).$$

**Solution:**