[gex92] Polygamma function: series representation, recurrence relation

(a) Extract the series representation of the Digamma function,

$$\psi(x) \doteq \frac{d}{dx} \ln \Gamma(x) = -\gamma + \sum_{n=0}^{\infty} \left[\frac{1}{n+1} - \frac{1}{x+n} \right] \quad : \; x > 0,$$

from the Weierstrass definition of the function $\Gamma(x)$.
(b) Infer the recurrence relation for the Digamma function,

$$\psi(x+1) = \psi(x) + \frac{1}{x},$$

from the familiar recurrence relation $\Gamma(x+1) = x\Gamma(x)$.

Solution: