[gex91] Definite integral via residue theorem III

(a) Evaluate the Fourier transform,

$$\tilde{f}(k) \doteq \int_{-\infty}^{\infty} dx\, e^{\imath kx} \underbrace{\frac{x}{x^2+2x+5}}_{f(x)},$$

of the function $f(x)$ separately for $k > 0$ and $k < 0$ via contour integrals.
(b) Check your results with the Mathematica command FourierTransform.

Hints: Use a semicircle in the upper half plane for the case $k > 0$ and a semicircle in the lower half plane for $k < 0$. Remember that the first (second) circle is traversed in the positive (negative) sense. There are variations in which Fourier transforms are defined. Check the definition used by Mathematica.

Solution: