[gex9] Area and circumference of an ellipse

The equation of an ellipse,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b > 0,$$

with semimajor axis a and semiminor axis b, is known as *level-curve* representation in differential geometry. Determine by integration the area A and the circumference C of the ellipse as functions of a and b. Recover from the general expressions the familiar results for both A and C in the limits $b = a$ of a circle, and $b = 0$ of a double line.

Solution: