

## [gex88] Residues of isolated singularities

Consider the three analytic functions with singularities at  $z = 0$ :

$$f_1(z) = \frac{(1 - \cos x)^2}{z^9}, \quad f_2(z) = \frac{\sin^3 z}{z^9}, \quad f_3(z) = z^2 e^{-1/z}.$$

- (a) Calculate the residue  $a_{-1}$  of that singularity for each function from the Laurent series.  
(b) For the two functions whose singularity is a pole, reproduce the residue from the limit,

$$a_{-1} = \lim_{z \rightarrow a} \frac{1}{(k-1)!} \frac{d^{k-1}}{dz^k} \left[ (z-a)^k f(z) \right]$$

- (c) For all three functions calculate the residue from the integral,

$$a_{-1} = \frac{1}{2\pi i} \oint_C dz f(z).$$

Hints: For part (a) use the `Series` command. For part (b) use the `D` command for derivatives and the `Limit` command. For part (c) integrate around a unit circle,  $z = e^{i\phi}$  with  $0 \leq \phi \leq 2\pi$ . Use the `Integrate` command for that purpose.

**Solution:**