

## [gex85] Laurent series of analytic functions I

The three complex functions,

$$f_1(z) = \frac{e^{2z}}{(z-1)^3}, \quad z = 1; \quad f_2(z) = \frac{z - \sin z}{z^3}, \quad z = 0; \quad f_3(z) = (z-3) \sin\left(\frac{1}{z+2}\right), \quad z = -2,$$

each have a singularity at the point indicated.

(a) Expand each function into a Laurent series of at least six terms at that point of singularity.

(b) Identify the type of singularity in each case (removable, essential, or pole of specific order).

Use the Mathematica command `Series` for assistance.

**Solution:**