[gex83] Poisson integrals for half plane in complex analysis

Consider a function $f(\zeta)$ with $\zeta = \xi + \imath\eta$ which is analytic for $\Im[\zeta] \geq 0$ and bounded, $|f(\zeta)| \leq K$. The Poisson integrals then infer the values at $\eta > 0$ (upper half plane) from the values at $\eta = 0$ (real axis).
Employ the Cauchy integral for a point z in the upper half plane and its conjugate $\bar{z}$ (in the lower half plane) to derive the following relation:

$$f(\zeta) = \frac{1}{\pi}\int_{-\infty}^{\infty} dx\, \frac{\eta f(x)}{(x-\xi)^2+\eta^2} \quad : \; \zeta = \xi + \imath\eta, \quad \eta > 0.$$

Solution: