

[gex80] Argument theorem in complex analysis

An elementary version of the argument theorem in complex analysis states that if $f(z)$ is analytic inside and on a simple closed curve C except for a pole of order p and a zero of order n , the following relation holds:

$$\frac{1}{2\pi i} \oint_C dz \frac{f'(z)}{f(z)} = n - p. \quad (1)$$

We recall that a pole of order p [zero of order n] means that we can express $f(z)$ in the form,

$$f(z) = \frac{F(z)}{(z - z_{\text{pole}})^p}, \quad \left[f(z) = (z - z_{\text{zero}})^n G(z) \right], \quad (2)$$

where $F(z)$ and $G(z)$ are analytic and nonzero inside and on C . Prove relation (1).

Solution: