

[gex8] Complete elliptic integrals: series expansion

(a) Derive the power series expansions of the two complete elliptic integrals as shown below by subjecting the integrands to a binomial expansion. Document every step taken.

$$K(m) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - m \sin^2 \theta}} = \frac{\pi}{2} \sum_{n=0}^{\infty} \binom{-1/2}{n} m^n,$$
$$E(m) = \int_0^{\pi/2} d\theta \sqrt{1 - m \sin^2 \theta} = \frac{\pi}{2} \sum_{n=0}^{\infty} \binom{1/2}{n} \binom{-1/2}{n} m^n.$$

(b) State the terms up to  $O(m^3)$  of each expansion explicitly.

**Solution:**