

[gex79] Application of Cauchy's theorem I

The function,

$$f_n(z) = \frac{1}{(z-a)^n}, \quad n \in \mathbf{N}, \quad a \in \mathbb{C},$$

is analytic in the complex plane except at the point $z = a$. Cauchy's theorem implies that the contour integral,

$$I_n \doteq \oint_C dz f_n(z),$$

vanishes if point a lies outside C and is independent of the shape of C if it lies inside.

(a) Use these facts to calculate (by hand) the value of I_n (i) for the case $n = 1$ and (ii) for $n > 1$ using a circle of radius r centered at point a .

(b) Confirm these values by evaluating the integral I_n using a circular contour centered at point 0 with (iii) radius $r < a$ and (iv) radius $r > a$. Employ Mathematica for this part.

Solution: