[gex76] Ellipsoidal coordinates

Ellipsoidal coordinates $\lambda,\ \mu,\ \nu$ are parametrized by the semi-axes $a > b > c > 0$ of a fictitious ellipsoid. They range is constrained by these parameters: $0 < \lambda < c^2 < \mu < b^2 < \nu < a^2$. The transformation relations between ellipsoidal and Cartesian coordinates is defined as follows:

$$\frac{x^2}{a^2-\lambda}+\frac{y^2}{b^2-\lambda}+\frac{z^2}{c^2-\lambda}=1, \quad \frac{x^2}{a^2-\mu}+\frac{y^2}{b^2-\mu}+\frac{z^2}{c^2-\mu}=1, \quad \frac{x^2}{a^2-\nu}+\frac{y^2}{b^2-\nu}+\frac{z^2}{c^2-\nu}=1.$$

(a) Bring these relations into the form $x(\lambda,\mu,\nu),\ y(\lambda,\mu,\nu),\ z(\lambda,\mu,\nu)$ using the Mathematica command Solve.
(b) Use the prescription outlined in [gmd2] to determine the scale factors h_λ, h_μ, h_ν for ellipsoidal coordinates, which enables us to state all differential operators explicitly.
(c) Demonstrate that the vectors $\mathbf{e}_\lambda, \mathbf{e}_\mu, \mathbf{e}_\nu$ form an orthonormal set.

Solution: