

### [gex75] Oblate spheroidal coordinates

Oblate spheroids are flattened ellipsoids of rotation, of (roughly) the shape of a lentil or convex lens. Oblate spheroidal coordinates,  $\xi \geq 0$ ,  $-\pi/2 \leq \eta \leq \pi/2$ ,  $0 \leq \phi < 2\pi$ , are tailored for this symmetry. They are related to Cartesian coordinates as follows:

$$x = a \cosh \xi \cos \eta \cos \phi, \quad y = a \cosh \xi \cos \eta \sin \phi, \quad z = a \sinh \xi \sin \eta.$$

- (a) Use the prescription outlined in [gmd2] to determine the scale factors  $h_\xi, h_\eta, h_\phi$  for oblate spheroidal coordinates, which enables us to state all differential operators explicitly.
- (b) Demonstrate that the vectors  $\mathbf{e}_\xi, \mathbf{e}_\eta, \mathbf{e}_\phi$  form an orthonormal set.
- (c) Use the Mathematica command `ParametricPlot` to visualize cross sections of the ellipsoids and hyperboloids similar to the plot shown in [gmd2].

**Solution:**