

[gex74] Prolate spheroidal coordinates

Prolate spheroids are elongated ellipsoids of rotation, of (roughly) the shape of an American football. Prolate spheroidal coordinates, $\xi \geq 0$, $-\pi/2 \leq \eta \leq \pi/2$, $0 \leq \phi < 2\pi$, are tailored for this symmetry. They are related to Cartesian coordinates as follows:

$$x = a \sinh \xi \sin \eta \cos \phi, \quad y = a \sinh \xi \sin \eta \sin \phi, \quad z = a \cosh \xi \cos \eta.$$

- (a) Use the prescription outlined in [gmd2] to determine the scale factors h_ξ, h_η, h_ϕ for prolate spheroidal coordinates, which enables us to state all differential operators explicitly.
- (b) Demonstrate that the vectors $\mathbf{e}_\xi, \mathbf{e}_\eta, \mathbf{e}_\phi$ form an orthonormal set.
- (c) Use the Mathematica command `ParametricPlot` to visualize cross sections of the ellipsoids and hyperboloids similar to the plot shown in [gmd2].

Solution: