

[gex73] Elliptic cylindrical coordinates

Elliptic cylindrical coordinates, $-\infty < u < \infty$, $0 \leq v < 2\pi$, $-\infty < z < \infty$, may be called, with equal justification, hyperbolic cylindrical coordinates. Their relation to Cartesian coordinates is

$$x = a \cosh u \cos v, \quad y = a \sinh u \sin v, \quad z = z.$$

- (a) Use the prescription outlined in [gmd2] to determine the scale factors h_u, h_v, h_z for elliptic cylindrical coordinates, which enables us to state all differential operators explicitly.
- (b) Demonstrate that the vectors $\mathbf{e}_u, \mathbf{e}_v, \mathbf{e}_z$ form an orthonormal set.
- (c) Establish transformation relations in the form $\rho(u, v)$ and $\phi(u, v)$ between elliptic and circular cylindrical coordinates.
- (d) Prove that the curves $u = \text{const}$ and $v = \text{const}$ are indeed sets of ellipses and hyperbolas, respectively.

Solution: