

[gex72] Paraboloidal coordinates

Whereas parabolic cylindrical coordinates [gex71] have an axis of continuous translational symmetry, paraboloidal coordinates with ranges $u \geq 0$, $v \geq 0$, $0 \leq \phi < 2\pi$ have an axis of continuous rotational symmetry, again chosen to be the z -axis. The transformation relations to Cartesian coordinates are

$$x = uv \cos \phi, \quad y = uv \sin \phi, \quad z = \frac{1}{2}(u^2 - v^2).$$

- (a) Use the prescription outlined in [gmd2] to determine the scale factors h_u, h_v, h_ϕ for parabolic coordinates, which enables us to state all differential operators explicitly.
- (b) Demonstrate that the vectors $\mathbf{e}_u, \mathbf{e}_v, \mathbf{e}_\phi$ form an orthonormal set.

Solution: