[gex7] First-order ODE: exact differential I

Consider the 1st-order ODE,

$$y' = \frac{3x^2 + y\cos x}{4y^3 - \sin x}.$$

The DSolve command of Mathematica readily yields four lengthy expressions $y(x)$, each with one integration constant as expected. The multiplicity indicates that the function is multiple-valued and/or that each solution only describes a segment of the general solution. A more compact expression of the general solution can be worked out once we recognize that the differential representation of the ODE is an exact differential.

(a) Express the above ODE as a differential.

(b) Confirm that the integrability condition is satisfied, which guarantees that the differential is of the form $dU(x,y) = 0$.

(c) Integrate the differential along a specific from $(0,0)$ to (x,y) to produce an implicit general solution $U(x,y) = c$. This expression is much more compact than the DSolve expressions.

(d) Use the Mathematica command ContourPlot to show curves for the solution using several values for the integration constant c.

(e) Reconstruct graphically one solution from part (d) by plotting the four DSolve solutions. Note that the integration constants are not expected to match.

Solution: gex7.nb