

[gex69] Gradient and Laplacian in rectilinear and cylindrical coordinates

Consider the scalar fields stated in Cartesian coordinates:

$$s_1 = x^2 + y^3 + z^4, \quad s_2 = \ln(x^2 + y^2), \quad s_3 = \frac{1}{\sqrt{x^2 + y^2 + z^2}}.$$

- (a) Express each field in cylindrical coordinates.
- (b) Calculate the gradient of each field using Cartesian and cylindrical coordinates.
- (c) Calculate the Laplacian of each using Cartesian and cylindrical coordinates.
- (d) Convert the vector ∇s_2 from Cartesian to cylindrical coordinates by using the transformation relations for the components and those for the unit vectors.

Hint: Use the Mathematica commands `Grad[v1[ρ, φ, z], {ρ, φ, z}, "Cylindrical"]` and equivalent for the application of differential operators.

Solution: