[gex68] Divergence and curl in rectilinear and cylindrical coordinates

Consider the following three vector fields stated in Cartesian coordinates:

$$\mathbf{v}_1 = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}, \quad \mathbf{v}_2 = -y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}}, \quad \mathbf{v}_3 = \ln(x^2 + y^2)\,\hat{\mathbf{k}}.$$

(a) Express each vector in cylindrical coordinates.
(b) Calculate the divergence of each vector using Cartesian and cylindrical coordinates.
(c) Calculate the curl of each vector using Cartesian and cylindrical coordinates.
(d) Convert the vector $\nabla \times \mathbf{v}_3$ from Cartesian to cylindrical coordinates by using the transformation relations for the components and those for the unit vectors.
Hints: Use the Mathematica commands Div[$\mathsf{v}_1[\rho, \phi, \mathsf{z}], \{\rho, \phi, \mathsf{z}\}$, "Cylindrical"] and equivalent for the application of differential operators.

Solution: