

## [gex61] Linear ODE solved via Laplace transform

Consider the following linear ODE with initial conditions specified:

$$y'' - 3y' + 2y = 2e^{-x}, \quad y(0) = 2, \quad y'(0) = -1.$$

- (a) Use the information supplied in [gmd8A] to express the Laplace transform of each term of this ODE. The Mathematica command `LaplaceTransform` is applicable to each term. The result is an algebraic equation for the function  $Y(s)$ , the Laplace transform of the desired solution  $y(x)$ .
- (b) Employ the `Solve` command to express  $Y(s)$  as a rational function.
- (c) Employ the command `InverseLaplaceTransform` to obtain the solution  $y(x)$  of the ODE as the sum of three exponential terms.
- (d) The three exponential terms are explained by the fact that the rational function  $Y(s)$  can be split into partial fractions of the most elementary kind,  $a/(b + s)$ , each of which is the Laplace transform of an exponential function. The command `Apart` does that for you. Check it out. In earlier times, this was the way to go.
- (e) The quickest way to solve the above ODE with Mathematica uses the `DSolve` command. Do it.

**Solution:**