

[gex56] Orthogonal family of functions I

Consider the analytic function $f(z) = ze^{-z}$.

(a) Express it in the form, $f(z) = u(x, y) + iv(x, y)$, where u and v are real, harmonic functions. The Mathematica command `ComplexExpand` does that efficiently.

The two one-parameter families of curves $u(x, y) = \alpha$, $v(x, y) = \beta$ are then mutually orthogonal at all points of intersection as explained in [gmd7].

(b) Plot curves from both families across the ranges $-1 < x < 3$, $-2 < y < 2$. choose the parameters α and β such that a compelling pattern of orthogonal intersections is present. The Mathematica command `ContourPlot` is a convenient tool for this purpose.

Solution: