[gex54] Harmonic functions and analytic function

Consider the two complex functions,

$$f_1(x, y) = \sin x \cosh y + i \cos x \cosh y, \quad f_2(x, y) = \sin x \sinh y + i \cos x \cosh y.$$

(a) Show that the real and imagninary parts of both functions are harmonic.
(b) Show that the real and imaginary parts of only one function satisfy the Cauchy-Riemann conditions, which makes it an analytic function.
(c) Express the analytic function in the form $f(z)$ and the other function in the form $f(z, \bar{z})$.

Solution: