[gex53] Euler's product representation of Gamma function

(a) Derive Euler's product representation of the Gamma function from the familiar integral representation:

$$\Gamma(x) \doteq \int_0^\infty dt\, e^{-t} t^{x-1} \quad \longrightarrow \quad \Gamma(x) = \lim_{n\to\infty} \frac{n!n^x}{x(x+1)(x+2)\cdots(x+n)}.$$

(b) Use the product representation to confirm the following well-known attributes of the Gamma function: $\Gamma(1) = 1$ and $\Gamma(x+1) = x\Gamma(x)$.
Hint: For part (a) use the identity $e^{-t} = \lim_{n\to\infty}(1-t/n)^n$ and sequential integrations by parts.

Solution: