## [gex52] Matrix operations XIII: eigenvectors of transition matrix

Transition matrices as used in Markov chain processes are, in general, asymmetric and have non-negative elements. The elements of each row must add up to unity. This has consequences for eigenvalues and eigenvectors. Here we consider the matrix,

$$\mathbf{P} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{7} & \frac{2}{7} & \frac{4}{7} \end{pmatrix}.$$

(a) Use the command N[Eigenvalues[ ]] to determine the three eigenvalues of $\mathbf{P}$. Verify that one eigenvalue is equal to 1 as must be the case for all transition matrices.
(b) Apply the command N[Eigenvectors[ ]] to the matrix $\mathbf{P}$ and its transpose to determine the (non-normalized) right eigenvectors and left eigenvectors.
(c) Verify that the right eigenvector associated with the unit eigenvalue has identical elements as must be the case.
(d) Normalize the left eigenvector associated with the unit eigenvalue by the sum of its elements. Then they represent the probabilities of a stationary state for the stochastic process in question.
(e) Verify that for the other two left eigenvectors the elements add up to zero.
(f) Demonstrate that each left eigenvector is orthogonal to two right eigenvectors and vice versa. They form a bi-orthogonal set.
Create a Mathematica notebook to carry out these tasks.

**Solution:**