[gex51] Matrix operations XII: eigenvectors of asymmetric matrix

Consider the constant, asymmetric matrix familiar from [gex42],

$$\mathbf{N} = \left(\begin{array}{rrr} 2 & 1 & 1 \\ 3 & 2 & 5 \\ 1 & 4 & 2 \end{array} \right).$$

(a) Use the command $\mathsf{N}[\mathsf{Eigenvalues}[~]]$ to determine the three eigenvalues of $\mathbf{N}.$

(b) Use the command N[Eigenvectors[]] to the matrix N and its transpose to determine the (non-normalized and complex) right eigenvectors and left eigenvectors.

(c) Use the Norm command to normalize the six eigenvectors.

(d) Demonstrate that each left eigenvector is orthogonal to two right eigenvectors and vice versa. They form a bi-orthogonal set.

Create a Mathematica notebook to carry out these tasks.

Solution: