[gex50] Matrix operations XI: eigenvectors of unitary matrix

Consider the constant, unitary matrix familiar from [gex44],

$$\mathbf{U} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & \frac{\imath}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & -\frac{\imath}{\sqrt{3}} \\ 0 & \frac{\imath}{\sqrt{3}} & -\frac{\imath}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}.$$

(a) Use the command N[Eigenvalues[ ]] to determine the four eigenvalues of $\mathbf{U}$.
(b) Verify that all four eigenvalues have unit norm.
(c) Reproduce these eigenvalues by applying the command NSolve to the characteristic polynomial Det[$\mathbf{U} - \lambda\mathbf{I}$], where $\mathbf{I}$ is the identity matrix.
(d) Use the command N[Eigenvectors[ ]] to determine the four (non-normalized and complex) eigenvectors of $\mathbf{U}$. Show that left and right eigenvectors are equivalent.
(e) Use the Norm command to normalize the four eigenvectors.
(f) Demonstrate that these eigenvectors then form an orthonormal set.
Create a Mathematica notebook to carry out these tasks.

Solution: