[gex49] Matrix operations X: eigenvectors of orthogonal matrix

Consider the constant, orthogonal matrix familiar from [gex43],

$$\mathbf{O} = \begin{pmatrix} \frac{6}{7} & \frac{2}{7} & \frac{3}{7} \\ \frac{3}{7} & -\frac{6}{7} & -\frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & -\frac{6}{7} \end{pmatrix}.$$

(a) Use the command N[Eigenvalues[ ]] to determine the three eigenvalues of $\mathbf{O}$.
(b) Verify that all three eigenvalues have unit norm.
(c) Reproduce these eigenvalues by applying the command NSolve to the characteristic polynomial Det$[\mathbf{O} - \lambda\mathbf{I}]$, where $\mathbf{I}$ is the identity matrix.
(d) Use the command N[Eigenvectors[ ]] to determine the three (non-normalized and complex) eigenvectors of $\mathbf{O}$. Show that left and right eigenvectors are identical.
(e) Use the Norm command to normalize the three eigenvectors.
(f) Demonstrate that these eigenvectors then form an orthonormal set.
Create a Mathematica notebook to carry out these tasks.

Solution: