[gex44] Matrix operations V: unitary matrix

Consider the constant square matrix,

$$\mathbf{U} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & \frac{\imath}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & -\frac{\imath}{\sqrt{3}} \\ 0 & \frac{\imath}{\sqrt{3}} & -\frac{\imath}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}.$$

Is it a unitary matrix?
(a) Demonstrate that $|\text{Det}[\mathbf{U}]| = 1$.
(b) Demonstrate that $\mathbf{U}^{-1} = \mathbf{U}^{\dagger}$.
(c) Construct a 3×3 unitary matrix $\tilde{\mathbf{U}}$ with $\text{Det}[\tilde{\mathbf{U}}] = \imath$ and vanishing diagonal elements.
Create a Mathematica notebook to carry out these tasks.

Solution: