[gex43] Matrix operations IV: orthogonal matrix

Consider the constant square matrix,

$$\mathbf{O} = \begin{pmatrix} \frac{6}{7} & \frac{2}{7} & \frac{3}{7} \\ \frac{3}{7} & -\frac{6}{7} & -\frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & -\frac{6}{7} \end{pmatrix}.$$

Is it an orthogonal matrix?
(a) Demonstrate that $|\mathrm{Det}[\mathbf{O}]| = 1$.
(b) Demonstrate that $\mathbf{O}^{-1} = \mathbf{O}^T$.
(c) Construct a 3×3 orthogonal matrix with negative determinant and vanishing diagonal elements.
Create a Mathematica notebook to carry out these tasks.

Solution: