

[gex40] Matrix operations I: matrix multiplication

Consider the two constant matrices,

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & 6 \\ -1 & 7 & -9 \\ -3 & -5 & 8 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 7 & -3 & 5 & 1 \\ 4 & -2 & 6 & 0 \\ -1 & 2 & -8 & -9 \end{pmatrix}.$$

- Calculate the product matrices $\mathbf{P} = \mathbf{AB}$ and $\mathbf{Q} = \mathbf{BA}$.
 - Express the matrix \mathbf{P} as the sum of a symmetric matrix \mathbf{P}_s and an antisymmetric matrix \mathbf{P}_a .
 - Do the same with matrix \mathbf{Q} .
 - Determine the determinants of \mathbf{P} , \mathbf{P}_s , \mathbf{P}_a , and \mathbf{Q} , \mathbf{Q}_s , \mathbf{Q}_a .
 - Determine the traces of \mathbf{P} , \mathbf{P}_s , \mathbf{P}_a and \mathbf{Q} , \mathbf{Q}_s , \mathbf{Q}_a .
 - Verify the identity, $\mathbf{Q}^2 = \mathbf{Q}_s^2 + \mathbf{Q}_s\mathbf{Q}_a + \mathbf{Q}_a\mathbf{Q}_s + \mathbf{Q}_a^2$.
- Create a Mathematica notebook to carry out these tasks.

Solution: