## [gex38] Representations of the Dirac delta function II

Consider the two functions,

$$\Delta_3(x,v) \doteq \frac{v}{\pi(1+v^2x^2)}, \quad \Delta_4(x,v) \doteq \frac{\sin(vx)}{\pi x},$$

as representations of the Dirac delta function $\delta(x)$ in the limit $v \to \infty$. Investigate to what extent the two attributes stated in [lln4],

$$\delta(x) = \begin{cases} 0 & : \; x \neq 0, \\ \infty & : \; x = 0, \end{cases} \qquad \int_{-\infty}^{+\infty} dx\, \delta(x) = 1.$$

are satisfied. Given the slow decay of these representations, the integrals

$$I_3(v) = \int_{-\infty}^{+\infty} dx\, f(x)\Delta_3(x-2,v), \quad I_4(w) = \int_{-\infty}^{+\infty} dx\, f(x)\Delta_4(x-2,v), \quad f(x) = x^2,$$

for large values of $v$ will not converge. Show that convergence toward the result $f(2) = 4$ can be restored by narrowing the range of integration judiciously as would be the case if $\delta(x-2)$ were substituted for $\Delta_3(x,v)$ or $\Delta_4(x,v)$ in the two integrals.

**Solution:**