[gex36] Vector divisions?

Considering the two kinds of multiplications of vectors – the dot product and the cross product – are there meaningful ways two divide by a vector? One way to put that question is by asking to find the vector $\mathbf{X}$ that solves the two equations,

$$\mathbf{A} \cdot \mathbf{X} = c, \quad \mathbf{A} \times \mathbf{X} = \mathbf{C},$$

for a given scalar c and a given vectors $\mathbf{C}$ and $\mathbf{A}$. Show that a solution only exists if $\mathbf{A} \perp \mathbf{C}$ and that the solution then reads

$$\mathbf{X} = \frac{\mathbf{C} \times \mathbf{A} + c\mathbf{A}}{\mathbf{A} \cdot \mathbf{A}}.$$

The quotient theorem in tensor analysis [gmd5] is an immensely useful elaboration of the concept of vector division.

Solution: