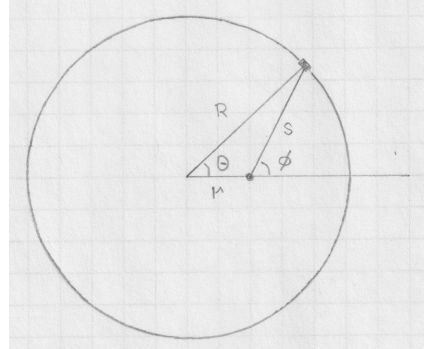


[gex34] Electric field of charged ring and elliptic integrals

Calculate the electric field of a uniformly charged ring of radius R and total charge Q at points along a radial line (in the plane of the ring). Show that the result can be expressed in terms of complete elliptic integrals [gmd4C] as follows:

$$E(r) = \frac{kQ}{\pi R^2 x(x^2 - 1)} \left[(x - 1) K \left(\frac{4x}{(1 + x)^2} \right) + (x + 1) E \left(\frac{4x}{(1 + x)^2} \right) \right], \quad x \doteq \frac{r}{R}, \quad k \doteq \frac{1}{4\pi\epsilon_0}.$$

Plot the expression on a scale that shows all its features. Infer from the general result asymptotic expressions for field points (i) near $r = 0$, (ii) near $r = R$, and (iii) at $r \gg R$.



Solution: