[gex32] Law of sines for spherical triangle

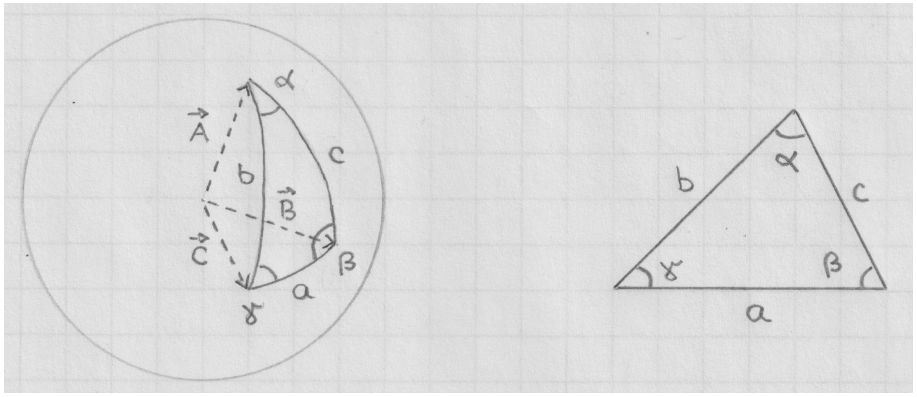
The law of sines for a triangle in a plane (image on the right) is a relation reviewed in [gmd1] involving the three sides $a,\ b,\ c$ and the opposite angles $\alpha,\ \beta,\ \gamma$:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \qquad \text{(plane)}.$$

Consider instead a triangle on the surface of a unit sphere (image on the left). The sides $a,\ b,\ c$ are segments of unit circle with opposite angles $\alpha,\ \beta,\ \gamma$. Show that the law of sines for triangles on the unit sphere reads,

$$\frac{\sin\alpha}{\sin a} = \frac{\sin\beta}{\sin b} = \frac{\sin\gamma}{\sin c} \qquad \text{(unit sphere)}.$$

Hint: Apply the identity, $(\mathbf{A}\times\mathbf{B})\times(\mathbf{A}\times\mathbf{C}) = (\mathbf{A}\cdot\mathbf{B}\times\mathbf{C})\mathbf{A}$, (found in [gmd1]) to the three radial unit vectors shown.



Solution: