

### [gex29] Expansion of vector in non-orthonormal basis

The expansion of a vector  $\mathbf{r}$  in the orthonormal basis  $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$  is constructed by projections via scalar products as  $\mathbf{r} = (\mathbf{r} \cdot \hat{\mathbf{i}})\hat{\mathbf{i}} + (\mathbf{r} \cdot \hat{\mathbf{j}})\hat{\mathbf{j}} + (\mathbf{r} \cdot \hat{\mathbf{k}})\hat{\mathbf{k}}$ . Show that this expansion can be generalized to a non-orthonormal basis  $\mathbf{a}, \mathbf{b}, \mathbf{c}$  as follows:

$$\mathbf{r} = (\mathbf{r} \cdot \mathbf{a}')\mathbf{a} + (\mathbf{r} \cdot \mathbf{b}')\mathbf{b} + (\mathbf{r} \cdot \mathbf{c}')\mathbf{c},$$

where

$$\mathbf{a}' = \frac{\mathbf{b} \times \mathbf{c}}{\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}}, \quad \mathbf{b}' = \frac{\mathbf{c} \times \mathbf{a}}{\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}}, \quad \mathbf{c}' = \frac{\mathbf{a} \times \mathbf{b}}{\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}},$$

are the reciprocal vectors as described in [gmd1].

**Solution:**