

[gex28] **Reciprocal vectors**

(a) Show that the three vectors,

$$\mathbf{a}' = \frac{1}{V} \mathbf{b} \times \mathbf{c}, \quad \mathbf{b}' = \frac{1}{V} \mathbf{c} \times \mathbf{a}, \quad \mathbf{c}' = \frac{1}{V} \mathbf{a} \times \mathbf{b}, \quad (1)$$

satisfy the orthonormality relations,

$$\mathbf{a}' \cdot \mathbf{a} = \mathbf{b}' \cdot \mathbf{b} = \mathbf{c}' \cdot \mathbf{c} = 1 \quad \mathbf{a}' \cdot \mathbf{b} = \mathbf{a}' \cdot \mathbf{c} = \mathbf{b}' \cdot \mathbf{a} = \mathbf{b}' \cdot \mathbf{c} = \mathbf{c}' \cdot \mathbf{a} = \mathbf{c}' \cdot \mathbf{b} = 0, \quad (2)$$

if  $V \doteq \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) > 0$  holds.

(b) Show that the original vectors  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$  are recovered if the  $\mathbf{a}'$ ,  $\mathbf{b}'$ ,  $\mathbf{c}'$  from (1) are substituted into the expressions,

$$\mathbf{a} = \frac{1}{v} \mathbf{b}' \times \mathbf{c}', \quad \mathbf{b} = \frac{1}{v} \mathbf{c}' \times \mathbf{a}', \quad \mathbf{c} = \frac{1}{v} \mathbf{a}' \times \mathbf{b}', \quad (3)$$

provided that  $v = 1/V$  holds.

**Solution:**