[gex27] Magnetic dipole field

Given the dipole term in the multipole expansion of the vector potential $\mathbf{A}(\mathbf{r})$ as derived in [lln12], derive the following expression for the magnetic dipole field $\mathbf{B}(\mathbf{r})$:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\,\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2}, \quad \hat{\mathbf{r}} \doteq \frac{\mathbf{r}}{r} \quad \Rightarrow\ \mathbf{B}(\mathbf{r}) \doteq \nabla\times\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\,\frac{[3\hat{\mathbf{r}}\,(\mathbf{m}\cdot\hat{\mathbf{r}}) - \mathbf{m}]}{r^3},$$

where $\mathbf{m}$ is the magnetic dipole moment and $\hat{\mathbf{r}}$ the unit vector pointing from the dipole (assumed localized) to the field point.
(a) Work out the solution by hand.
(b) Work out the solution in a Mathematica notebook. A vector is List. Its magnitude is Norm. The cross product is Cross, The curl is Curl.
(c) What is the relative orientation of $\mathbf{B}$ and $\mathbf{m}$ (i) if $\hat{\mathbf{r}}$ and $\mathbf{m}$ are parallel, (ii) if $\hat{\mathbf{r}}$ and $\mathbf{m}$ are perpendicular, (iii) if $\hat{\mathbf{r}}$ and $\mathbf{m}$ are antiparallel.

Solution: