

[gex16] Trigonometric relations made transparent by complex variables

(a) By using Euler's formula,  $e^{i\phi} = \cos \phi + i \sin \phi$ , prove the following trigonometric relations:

(i)  $\sin(2\phi) = 2 \sin \phi \cos \phi$ ,

(ii)  $\cos(2\phi) = \cos^2 \phi - \sin^2 \phi$ ,

(iii)  $\sin^3 \phi = \frac{3}{4} \sin \phi - \frac{1}{4} \sin(3\phi)$ .

(b) Use Mathematica to verify that these identities are true.

**Solution:**