[gex15] First-order ODE: Bernoulli type

The Mathematica DSolve command has no trouble producing two general solutions of the nonlinear $1^{\mathrm{st}}\text{-}\mathrm{order}$ ODE,

$$xy' + y = xy^3$$

Why are there two independent solutions, each with one integration constant as expected? The source of this doubling becomes clear when we realize that this ODE is of the Bernoulli type and then analyze it according to the recipe outlined in [gmd10]. Do it.

Solution: