

[gex14] First-order ODE: homogeneity

The 1st-order ODE,

$$y' = \frac{2x^3 + y^3}{3xy^2},$$

is nonlinear, which, in general, signals complexity. However, a closer look reveals that it is homogeneous, which means that it can be solved by separation of variables as explained in [gmd10-A].

(a) Follow the instructions given there and show that the implicit one-parameter solution can be brought into the form,

$$y^3 - x^3 = cx.$$

(b) The Mathematica command `DSolve` applied to the above ODE produces three explicit one-parameter solutions, each one representing a segment of the general solution. Show that from each explicit solution the implicit solution can be inferred.

(c) Use the command `ContourPlot` with plot ranges $-2 < x, y < 2$ to show a graph of the implicit solution for $c = -2$. Use the command `Plot` to show the three explicit solutions in the same format and different colors. Superimpose the curves by use of the command `Show` to demonstrate that the solutions are equivalent.

Solution: