

[gex12] First-order ODE: integrating factor I

Consider the two 1st-order ODEs,

$$y' = \frac{y + x^4}{x}, \quad z' = \frac{z - x^3 - xz^2}{x}.$$

- (a) In both cases, the Mathematica DSolve command produces compact general solutions featuring one integration constant as expected. Verify that this is indeed the case.
- (b) Write both ODEs as differentials and show that they are inexact.
- (c) Show that the first differential can be made exact by the integrating factor $1/x^2$ and the second by the integrating factor $1/(x^2 + z^2)$.
- (d) The solution of both exact differentials are readily integrated along a specific path to produce the general solutions. Do it.
- (e) Verify that the solutions obtained via differentials made exact by integrating factors are equivalent to the solutions found by Mathematica.

Solution: