[gex116] Second-order ODE: reduction to first order ODE III

Consider the 2nd-order ODE for the function $y(x)$,

$$y''(x) + \sqrt{y(x)} = 0,$$

which is amenable to a reduction into a 1st-order ODE for the inverse function $x(y)$.

(a) Solve the original 2nd-order ODE for initial conditions $y(0) = y'(0) = 0$ via the DSolve command of Mathematica.

(b) Rewrite this 2nd-order ODE as a 1st-order ODE for the variable $s(y) \doteq x'(y)$.

(c) Solve this 1st-order ODE via the DSolve command.

(d) Use the Integrate command to determine $x(y)$ and the Solve command to determine $y(x)$ for comparison with the result of part (a).

Solution: