

[gex115] Linear inhomogeneous ODE: undetermined parameters II

Consider the linear, inhomogeneous 2nd-order ODE,

$$y'' - 6y' + 25y = 64xe^{-x}.$$

- (a) Examine the characteristic polynomial of the homogeneous ODE and identify if the solution is of case #1 (real roots), case #2 (complex roots), or case #3 (repeated roots) [gam8].
- (b) Use the DSolve command of Mathematica to find the complementary solution $Y_c(x)$, which is the general solution of the homogeneous ODE and features two integration constants.
- (c) Use the DSolve command to find the general solution $y(x)$ of the inhomogeneous ODE.
- (d) Extract the particular solution $Y_p(x) = y(x) - Y_c(x)$ from the results of parts (a) and (b).
- (e) Determine $Y_p(x)$ by the method of undetermined constant parameters [gam8] and reconcile the difference with the expression found in part (c).

Solution: